Fire Hose

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1 Introduction

While watering your garden you may have observed that a water hose can start to oscillate wildly once released. Readers familiar with astrophysics may also be acquainted with the fire hose instability, which causes an elongated galaxy to buckle along its long axis [1]. The analogy with the phenomenon you can observe in your back yard gave name to the astrophysics version. The mechanisms behind this behaviour will be discussed throughout this paper. The aim is to explain when this characteristic motion starts (onset) and to give a brief insight into the dynamics of the phenomenon. The derived model is in good agreement with the experimental results.

Work on similar topics has previously been conducted by Hansen [2], Doaré and Langre [3] and several others. However, their work is not specifically focused on the above problem. In addition, this paper provides a more intuitive and conceptual understanding of the phenomenon.

This problem has also been released for the International Young Physicists' Tournament (IYPT) 2013 in Taiwan [4]. The exact task reads:

"Fire Hose: Consider a hose with a water jet coming from its nozzle. Release the hose and observe its subsequent motion. Determine the parameters that affect this motion."

2 Theory

We will first explain how the motion starts (onset) and then investigate the dynamics of this phenomenon.

2.1 Onset

First, it is important to notice that the diameter of the hose is commonly larger than the diameter of the nozzle. As a consequence a change of momentum occurs, as the fluid must flow faster through a narrower cross section. This in return then leads to a force, which we will denote with F_r, acting on the hose (in opposite direction of the flow).

Let p denote the momentum of the fluid, v its velocity and $Q = \frac{dV}{dt} = \frac{1}{\rho}\frac{dm}{dt}$ the flow rate. With that we get

$$\frac{dp}{dt} = \frac{d(mv)}{dt} = \frac{dm}{dt}v + m\frac{dv}{dt} = \rho Q v + \sum_i F_{i,\,ext} = 0. \tag{1}$$

As the flow rate is a constant and the external force is equal to F_r we arrive at

$$|dF_r| = \rho Q dv \Rightarrow |F_r| = \rho Q(v_n - v_i) = \rho Q^2 \left(\frac{1}{A_n} - \frac{1}{A_i}\right), \tag{2}$$

where the index n refers to the nozzle and i to the hose and A is the respective cross sectional area. Note that we have used the relation $Q = vA$.

The force needed for buckling is well documented in literature (see for example [5]). In our case one end is fixed and the other is free to move laterally. Then the buckling constant is known to be $K = 2$ so that we get

$$F_B = \frac{\pi^2 Y I}{(Kl)^2} = \frac{\pi^2 Y I}{4l^2}, \tag{3}$$

with Y being the modulus of elasticity, I the area moment of inertia and l the length of the hose.

When the hose is lying on a flat substrate we also observe a friction force F_F. For our case we assume the the friction force is a constant c (static friction). Together with equation (2) and (3) we can then set up a relation for the motion onset:

$$F_r = F_F + F_B \Leftrightarrow \rho Q^2 \left(\frac{1}{A_n} - \frac{1}{A_i}\right) = c + \frac{\pi^2 Y I}{4l^2}. \tag{4}$$

Equation (4) is compared to the experimental results in figure 3a.

2.2 Oscillation

As the reader may have already observed himself, the hose performs an oscillatory motion. It turns out that once the onset occurs, one first observes a harmonic oscillation. When increasing the flow rate and/or the length, one will then observe higher modes. In this paper only the first mode will be derived in detail and a qualitative explanation for the mode jumps will be presented.

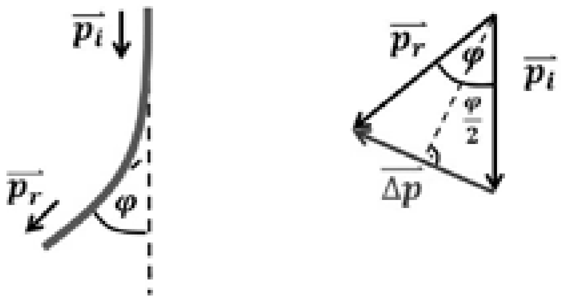

Figure 1: *Once the hose buckled there exists a curve segment along the hose. This forces the fluid to change its direction resulting in a restoring force acting on the hose. This causes the hose to oscillate.*

As soon as the hose buckles it will have a curvature. This forces the fluid to change direction along its path. Thus the fluid will push the hose back and act as a restoring force. This force drives the oscillation (see figure 1 and 3b). Knowing this we can quantify the restoring force F_k arising due to this momentum change:

$$F_k = 2\sin\left(\frac{\varphi}{2}\right)\frac{dp}{dt} = 2\sin\left(\frac{\varphi}{2}\right)\rho Q v = 2\sin\left(\frac{\varphi}{2}\right)\frac{\rho Q^2}{A} \tag{5}$$

Now torque balance can be applied. Note that J stands for the moment of inertia.

$$-\frac{d^2\varphi}{dt^2}J = \frac{l}{2}F_k = l\sin\left(\frac{\varphi}{2}\right)\rho\frac{Q^2}{A} \tag{6}$$

Solving the ODE using small angle approximation yields

$$\frac{d^2\varphi}{dt^2} + \frac{l\rho Q^2}{2AJ}\varphi = 0 \Rightarrow T = 2\pi\sqrt{\frac{2AJ}{l\rho Q^2}} \Rightarrow T \propto \frac{1}{Q} \wedge T \propto l \tag{7}$$

For the second proportionality we have used the fact that the moment of inertia for a tube with length l and a constant linear mass density is proportional to l^3. The second proportionality from equation (7) is compared to the experimental data in figure 4b.

We now want to investigate the mode jumps. Let us first consider the case of a hose of constant length l. The hose is fixed on one end and is allowed to move freely on the other. As for standing waves on a string we get

$$\text{mode}\,0:\; l = \frac{\lambda_0}{4} \Rightarrow f_0 = \frac{c}{4l}, \qquad \text{mode}\,i:\; l = \frac{(2i+1)\lambda_i}{4} \Rightarrow f_i = \frac{c}{4l}(2i+1) = f_0\,(2i+1) \tag{8}$$

for the frequency f with c denoting the propagation speed of the wave and λ the wavelength.

We now consider the case of a fixed flow rate. As seen in section 2.1 one can find the minimal length at which the hose starts to oscillate for a fixed length. Let l_0 denote this onset length. When gradually increasing the hose length we observe an interesting feature: At $3l_0$, $5l_0$, $7l_0$, ... we observe a mode jump. A schematic representation can be seen in figure 2.

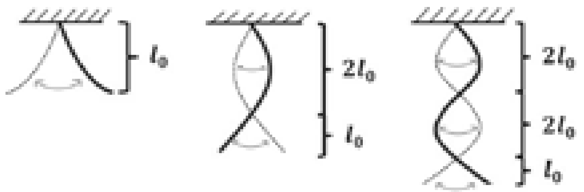

Figure 2: *Schematical representation of the higher modes when keeping the flow rate constant.* l_0 *represents the critical length at which the onset occurs for a given flow rate.*

The corresponding dominating frequency after a mode jump can again be derived easily using standing waves theory. We define the wavelength of the first mode λ_0 as $4\,l_0$. Let $l_i = (2i+1)l_0$ be the length of the hose when it jumps into the (i+1)-th mode. f_i^j symbolizes the (j+1)-th multiple of the fundamental frequency f_i^0 of the (i+1)-th mode. λ_i denotes the wavelength belonging to f_i^j.

$$l_1 = 3l_0 = \frac{3\lambda_0}{4} \Rightarrow \lambda_0 = \frac{4}{3}l_1 \Rightarrow f_1^1 = \frac{3c}{4l_1} = f_0^0 \Rightarrow f_1^0 = \frac{c}{12l_0} = \frac{f_0^0}{3} \tag{9}$$

The last implication follows from $l_0 = \frac{\lambda}{4} \Leftrightarrow f_0^0 = \frac{c}{4l_0}$ and equation (8). For the general case we again get

$$l_i = (2i+1)l_0 = \frac{(2i+1)\lambda_i}{4} \Rightarrow \lambda_i = \frac{4}{2i+1}l_i \Rightarrow f_i^i = f_0^0 \Rightarrow f_i^0 = \frac{c}{(2i+1)4l_0} = \frac{f_0^0}{2i+1}. \quad (10)$$

3 Materials and Methods

The experiments were conducted using two hoses of different elasticity. The parameters that were varied are the flow rate and the length of the hose. "Logger Pro" was used to track the motion of the hose. The fire hose motion was observed on a low friction plate.

4 Experimental Results

The errors shown in the plots are estimated errors.

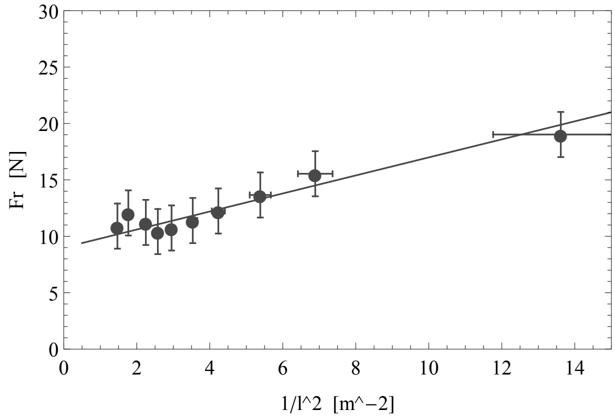

Figure 3a: *Experimental verification of equation (4). The offset due to friction is a fit parameter. The used hose has the property* $YI = (0.32 \pm 0.02)Nm^2$.

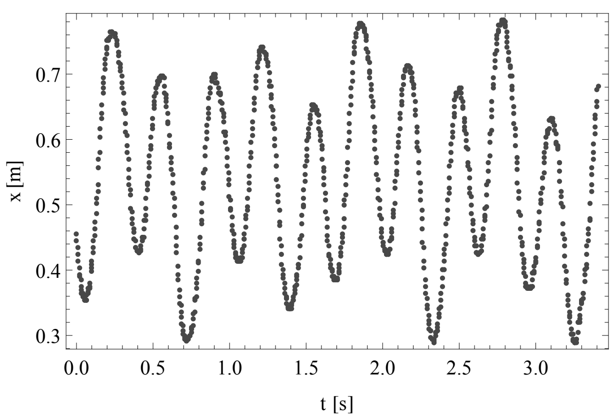

Figure 3b: *Tracking data of the hose obtained with "Logger Pro". Typical oscillation movement the hose performs.*

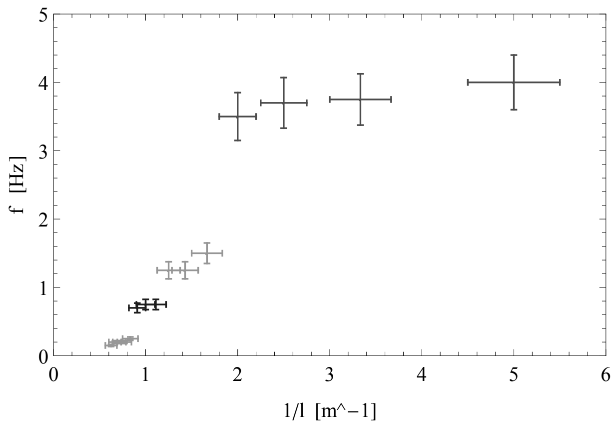

Figure 4a: *Observed fundamental frequencies* f_i^0 *depending on different lengths l for a constant flow rate. One can clearly see the characteristic mode jumps discussed in 2.2. Note that the data point on the top right corresponds to the onset.*

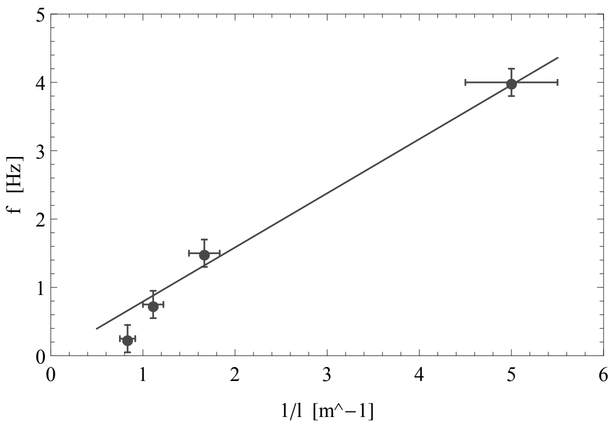

Figure 4b: *Data and linear fit function of the fundamental frequency* f_i^0 *immediately after a mode jump. Note that the point on the top right is* f_0^0. *The frequency after the jump and the length at which it occurs can be estimated using equation (10).*

5 Discussion

A robust model describing the onset was derived in section 2.1. The onset is explained through the diameter change between hose and nozzle, but you may still observe this motion even without a nozzle. This is a consequence of hoses already having an initial curvature due to its storage method (rolled up). However, if a nozzle is present the first effect will be dominant.

As previously discussed one eventually will observe a mode jump when one takes a hose of fixed length and gradually increases the flow rate. Hence, one can also experimentally verify the relation $f \propto Q$. However, due to limitations (most commonly flow rate limited by the onset as lower bound and bursting of the hose as upper bound) it is not possible to observe several jumps with this technique. So it is easier to keep the flow rate constant and increase the length instead.

A more detailed investigation of the dynamics and in particular a precise equation of motion could be derived. For example, all damping terms in equation (7) were neglected as it was only used to obtain an expression for the frequency. A further investigation on this non-linear, damped, driven oscillation and the subsequent mode jumps is highly intriguing, but clearly lies beyond the scope of this paper.

6 Conclusion

In section 2.1 we described the basic mechanism behind the onset using force balance. It is shown that the oscillation starts once the hose buckles. The model is in good agreement with the experimental data. In addition, a conceptual and intuitive explanation for the mode jumps is presented. The predicted frequency jumps are experimentally observed.

7 References

[1] D. Merritt, *Dynamics of Elliptical Galaxies*, 1993, Science, vol. 259.

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[3] O. Doaré, E. d. Langre, *The Flow-Induced Instability of long hanging Pipes*, 2002: European Journal of Mechanics - A/Solids, vol. 21.

[4] *Problems for the IYPT 2013*, http://archive.iypt.org/problems/.

[5] J. M. Gere, *Mechanics of Materials 6th edition*, 2004: Thomson Learning Inc., p. 773.